A OPTIMAL COLOR MODULATION DERIVATION

Given an ellipse constraint function

$$\begin{cases} \mathcal{E}(\mathbf{x}) = \left(\frac{x_1 - t_1}{a_1}\right)^2 + \left(\frac{x_2 - t_2}{a_2}\right)^2 - 1 = 0 \\ x_3 = t_3, \end{cases} \tag{16}$$

and a power cost function

$$\mathcal{P}(\mathbf{x}) = p_1 x_1 + p_2 x_2 + p_3 x_3 + p_{circ}, \tag{17}$$

we may solve the power optimizing $\mathbf{x}^*$ via the method of Lagrange multipliers.

First, we notice that x_3 should not change. Intuitively, this effectively reduces the dimensionality of the optimization onto the plane $x_3 = t_3$. Formally, we may rewrite the constraint and power functions in terms of a 2-dimensional variable $\mathbf{y} = (y_1, y_2) = (x_1, x_2)$:

$$\mathcal{E}(\mathbf{y}) = \left(\frac{y_1 - t_1}{a_1}\right)^2 + \left(\frac{y_2 - t_2}{a_2}\right)^2 - 1 = 0, \tag{18}$$

and

$$\mathcal{P}(\mathbf{y}) = p_1 y_1 + p_2 y_2 + const. \tag{19}$$

The minimizing vector $\mathbf{y}^*$ satisfies the condition that the gradients of $\mathcal{E}$, and $\mathcal{P}$ are co-linear. So the system of equations we need to solve for $\mathbf{y}^*$ is

$$\begin{cases} \nabla\mathcal{E}(\mathbf{y}^*) = \phi\nabla\mathcal{P}(\mathbf{y}^*) \\ \mathcal{E}(\mathbf{y}^*) = 0, \end{cases} \tag{20}$$

for some scalar constant ϕ.

Computing the gradients, we get

$$\begin{cases} \frac{2}{a_1}\frac{y_1^* - t_1}{a_1} = \phi p_1 \\ \frac{2}{a_2}\frac{y_2^* - t_2}{a_2} = \phi p_2 \\ \left(\frac{y_1^* - t_1}{a_1}\right)^2 + \left(\frac{y_2^* - t_2}{a_2}\right)^2 - 1 = 0. \end{cases} \tag{21}$$

Finally, we solve for $\mathbf{y}^*$ using this system of equations to get the optimal color, $\mathbf{x}^*$:

$$\begin{aligned} x_1^* &= \frac{p_1 a_1^2}{\sqrt{p_1^2 a_1^2 + p_2^2 a_2^2}} \\ x_2^* &= \frac{p_2 a_2^2}{\sqrt{p_1^2 a_1^2 + p_2^2 a_2^2}} \\ x_3^* &= t_3. \end{aligned} \tag{22}$$

B INDIVIDUAL PARTICIPANT DATA FOR PILOT PERCEPTUAL USER STUDY

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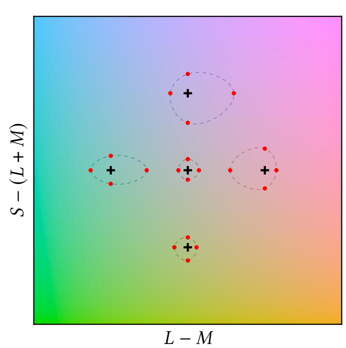
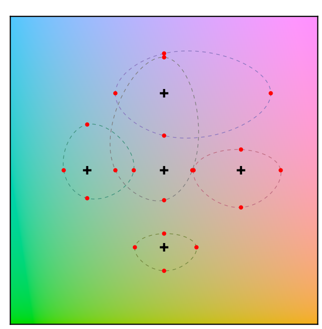
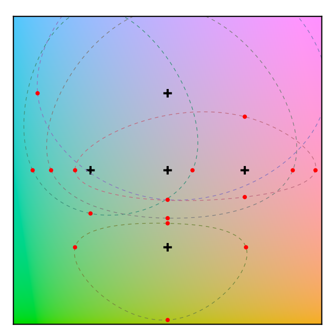
subject_01			

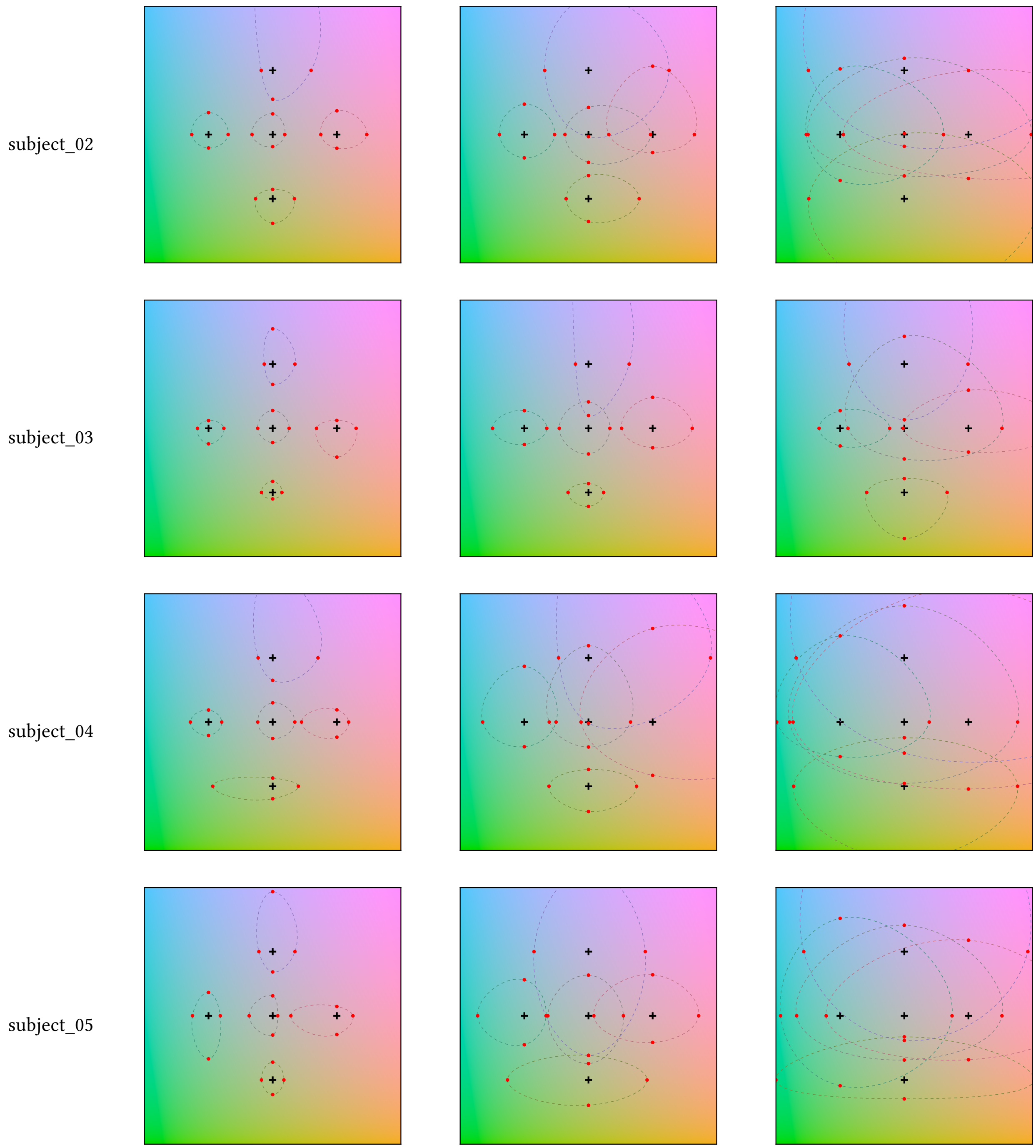


Table 3. Pilot Perceptual Study Threshold Data